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Cho Abelian Decomposition of Monopole-Antimonopole Pair Gauge Potentials

(Penghuraian Abelan Cho kepada Keupayaan Tolok Pasangan Monokutub-Antimonokutub)

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ABSTRACT

Recently we have reported on standard MAP and generalized Jacobi Elliptic monopole-antimonopole pair (MAP) solutions of the SU(2) Yang-Mills-Higgs model. Here we apply Cho Abelian decomposition to the gauge potential of these MAP solutions. It is shown that the point singularities at the locations of the monopole (antimonopole), that comes from the restricted part, are removed by the unrestricted valence potential. We also consider the effect of decomposition upon energy and magnetic charge density for the cases of standard MAP and generalized Jacobi elliptic MAP solutions, under the conditions of vanishing ($\lambda = 0$) and non vanishing ($\lambda = 1$) Higgs potential.

Keywords: Cho Abelian decomposition; monopole; Yang-Mills-Higgs

ABSTRAK

Sebelum ini, kami telah melaporkan penyelesaian MAP-piawai dan pasangan monokutub-antimonokutub (MAP) Jacobi Eliptik umum kepada model SU(2) Yang-Mills-Higgs. Di sini kami menggunakan kaedah penghuraian Abelian Cho ke atas keupayaan tolok penyelesaian tersebut. Kami menunjukkan titik tak-terhingga di lokasi monokutub (antimonokutub) yang berasal daripada bahagian terhad boleh dipadamkan oleh keupayaan valens. Kami juga mengambil kira kesan penghuraian ke atas tenaga dan ketumpatan cas magnet bagi kes MAP-piawai dan penyelesaian MAP Jacobi eliptik, dalam keadaan keupayaan Higgs lenyap ($\lambda = 0$) dan tidak lenyap ($\lambda = 1$).

Kata kunci: Monokutub; penghuraian Abelian Cho; Yang-Mills-Higgs

INTRODUCTION

The SU(2) Yang-Mills-Higgs (YMH) field theory in 3+1 dimensions, with the Higgs field in the adjoint representation possesses large varieties of magnetic monopole solutions. The most famous monopole solution is the ‘t Hooft-Polyakov monopole solution (‘ t Hooft 1974; Polyakov 1974) with finite energy and it is invariant under the U(1) subgroup of the local SU(2) gauge group. The ‘t Hooft-Polyakov monopole is a numerical solution and possesses non zero Higgs mass and self-interaction. The analytic form of the ‘t Hooft-Polyakov solution is reported by Bogomol’nyi (1976) and Prasad and Sommerfield (1975) under the Bogomol’nyi-Prasad-Sommerfield (BPS) limit.

The YMH field theory with a unit magnetic charge and finite energy is generally spherically symmetric (Forgács et al. 1981a, 1981b; Prasad 1981; Prasad & Rossi 1981; Rebbi & Rossi 1980; Ward 1981; Weinberg & Guth 1976), whereas multimonopole configurations with magnetic charges greater than unity cannot possess spherical symmetry (Teh & Wong 2005) but at most axial symmetry (Teh et al. 2010). The exact monopole and multimonopoles solutions (Bogomol’nyi 1976; Forgács et al. 1981a, 1981b; Prasad 1981; Prasad & Rossi 1981; Prasad & Sommerfield 1975; Rebbi & Rossi 1980; Ward 1981) exist only in the BPS limit. Outside the BPS with non-vanishing Higgs potential, only numerical solutions are known.

In Teh and Wong (2005), we have shown that the YMH model actually possesses more exact multimonopole-antimonopole configurations in the BPS limit. Recently we have also shown that the ‘t Hooft-Polyakov solutions has a Jacobi elliptic generalization (Teh et al. 2010) and the Kleihaus-Kunz MAP solutions (Kleihaus & Kunz 2000) can also include generalized Jacobi elliptic MAP functions (Teh et al. 2012). Magnetic monopole solutions with half-integer charges are also reported (Ng et al. 2012; Teh et al. 2012).

Cho (1980,1981) and Cho et al. (2008) have shown that it is possible to decompose the gauge potential into two parts, the restricted and the valence part. The restricted potential has a built-in electric-magnetic duality and possesses maximal Abelian subgroup H of gauge group G. The unrestricted part consists of the valence potential of G/H and it transforms covariantly under G. It is also stressed that the restricted gauge theory, despite made of only the restricted potential which has much less physical degrees of freedom, still retains the full gauge invariance.

The full non-Abelian gauge theory can be recovered simply by adding the valence part to the restricted part. Hence the non-Abelian gauge theory can be interpreted as a restricted gauge theory which has the valence potential as the gauge covariant source (Cho 1980,1981; Cho et al. 2008). The decomposition of full gauge theory into restricted part plays an important role to establish the Abelian dominance in non-Abelian dynamics. This is important in providing insights for proving the monopole condensation and confinement of color in QCD (Cho & Pak 2002; Cho et al. 2002; Faddeev & Niemi 1999a,1999b).

In this paper, we introduced Cho Abelian decomposition to the Yang-Mills gauge potential of the standard MAP and Jacobi elliptic MAP solutions (Teh et al. 2012) and studied the effect of decomposition upon the gauge potential profile functions, total energy and magnetic charge density. It is found that the restricted part of the gauge potential possesses two singularities at the locations of the monopole and antimonopole, respectively. Similar to the case of introducing the valence part to the restricted potential for the Wu-Yang-like monopole, where the singularity of the Wu-Yang-like monopole is removed and then producing a 't Hooft-Polyakov monopole with finite energy, the valence potential possesses two singularities (with opposite magnitude) and remove the singularities in the restricted profile functions, rendering a smooth Yang-Mills gauge potential.

THE SU(2) YMH THEORY AND EXACT ASYMPTOTIC SOLUTION

The SU(2) YMH Lagrangian in 3+1 dimensions with non vanishing Higgs potential is:

$$L = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{2}D^\mu\Phi^a D_\mu\Phi^a - \frac{1}{4}\lambda\left(\Phi^a\Phi^a - \frac{\mu^2}{\lambda}\right)^2. \quad (1)$$

Here the Higgs field mass is m and the strength of the Higgs potential is λ which are constants. The vacuum expectation value of the Higgs field is $\xi = \mu\sqrt{\lambda}$. The Lagrangian (1) is gauge invariant under the set of independent local SU(2) transformations at each space-time point. The covariant derivative of the Higgs field and the gauge field strength tensor are given respectively by:

$$\begin{aligned} D_\mu\Phi^a &= \partial_\mu\Phi^a + g\varepsilon^{abc}A^b_\mu\Phi^c, \\ F^a_{\mu\nu} &= \partial_\mu A^a_\mu - \partial_\nu A^a_\mu + g\varepsilon^{abc}A^b_\mu A^c_\nu, \end{aligned} \quad (2)$$

where g is the gauge field coupling constant. The metric used is $g_{uv} = (- + + +)$. The SU(2) internal group indices and the space-time indices $a, b, c = 1, 2, 3$ are $\mu, v, \alpha = 0, 1, 2$ and 3 in Minkowski space. The equations of motion that follow from the Lagrangian (1) are:

$$\begin{aligned} D^\mu F^a_{\mu\nu} &= \partial_\mu F^a_{\mu\nu} + g\varepsilon^{abc}A^{b\mu}F^c_{\mu\nu} = g\varepsilon^{abc}\Phi^b D_\nu\Phi^c, \\ D^\mu D_\mu\Phi^a &= \lambda\Phi^a\left(\Phi^b\Phi^b - \xi^2\right). \end{aligned} \quad (3)$$

In the limit of vanishing μ and λ, the Higgs potential vanishes and self-dual solutions can be obtained by solving the first order partial differential Bogomol'nyi equation, $B^a_i \pm D_i\Phi^a = 0$, where $B^a_i = -1/2\varepsilon_{ijk}F^a_{jk}$.

The electromagnetic field tensor proposed by 't Hooft ('t Hooft 1974) upon symmetry breaking is:

$$F_{\mu\nu} = \hat{\Phi}^a F^a_{\mu\nu} - \frac{1}{g}\varepsilon^{abc}\hat{\Phi}^a D_\mu\hat{\Phi}^b D_\nu\hat{\Phi}^c = G_{\mu\nu} + H_{\mu\nu}, \quad (4)$$

where $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $H_{\mu\nu} = 1/g\varepsilon^{abc}\hat{\Phi}^a\partial_\mu\hat{\Phi}^b\partial_\nu\hat{\Phi}^c$ are the gauge part and the Higgs part of the electromagnetic field, respectively. Here $A_\mu = \hat{\Phi}^a A^a_\mu$, the Higgs unit vector, $\hat{\Phi}^a = \Phi^a/|\Phi|$, and the Higgs field magnitude $|\Phi| = \sqrt{\Phi^a\Phi^a}$. Hence the decomposed magnetic field is $B_i = -1/2\varepsilon_{ijk}F_{jk} = B^G_i + B^H_i$, where B^G_i and B^H_i are the gauge part and Higgs part of the magnetic field respectively. The net magnetic charge of the system is:

$$M = \frac{1}{4\pi}\int \partial^i B_i d^3x = \frac{1}{4\pi}\oint d^2\sigma_i B_i. \quad (5)$$

The topological magnetic current, $k_\mu = \frac{1}{8\pi}\varepsilon_{\mu\nu\rho\sigma}\varepsilon_{abc}\partial^\nu\hat{\Phi}^a\partial^\rho\hat{\Phi}^b\partial^\sigma\hat{\Phi}^c$, is also the topological current density of the system (Manton 1977). Hence the corresponding conserved topological magnetic charge is,

$$\begin{aligned} M_H &= \frac{1}{g}\int d^3x k_0 = \frac{1}{8\pi g}\int \varepsilon_{ijk}\varepsilon^{abc}\partial_i\left(\hat{\Phi}^a\partial_j\hat{\Phi}^b\partial_k\hat{\Phi}^c\right)d^3x \\ &= \frac{1}{8\pi}\oint d^2\sigma_i\left(\frac{1}{g}\varepsilon_{ijk}\varepsilon^{abc}\left(\hat{\Phi}^a\partial_j\hat{\Phi}^b\partial_k\hat{\Phi}^c\right)\right) \\ &= \frac{1}{4\pi}\oint d^2\sigma_i B^H_i. \end{aligned} \quad (6)$$

The magnetic charge M_H is the total magnetic charge of the system if and only if the gauge field is non singular (Arafune et al. 1975). If the gauge field is singular and carries Dirac string monopoles M_G,

$$\begin{aligned} M_G &= -\frac{1}{8\pi}\oint d^2\sigma_i\varepsilon_{ijk}\left(\partial_j A_k - \partial_k A_j\right) \\ &= \frac{1}{4\pi}\oint d^2\sigma_i B^G_i. \end{aligned} \quad (7)$$

then the total magnetic charge of the system is just $M = M_G + M_H$.

For a non BPS solution, the dimensionless total energy is:

$$\begin{aligned} E &= \frac{g}{8\pi\xi}\int\left\{B^a_i B^a_i + D_i\Phi^a D_i\Phi^a + \frac{\lambda}{2}\left(\Phi^a\Phi^a - \xi^2\right)^2\right\} \\ & d^3x = \int \varepsilon\, dr\, d\theta, \end{aligned} \quad (8)$$

where ε is the energy density which is given by:

$$\varepsilon = \frac{g}{4\xi}r^2\sin\theta\left\{B^a_i B^a_i + D_i\Phi^a D_i\Phi^a + \frac{\lambda}{2}\left(\Phi^a\Phi^a - \xi^2\right)^2\right\}. \quad (9)$$

In the electrically neutral BPS limit with vanishing Higgs potential, the energy is a minimum, (Teh et al. 2010)

$$\begin{aligned} E_{min} &= \mp\int \partial_i\left(B^a_i\Phi^a\right)d^3x + \int\frac{1}{2}\left(B^a_i \pm D_i\Phi^a\right)d^3x \\ &= \mp\int \partial_i\left(B^a_i\Phi^a\right)d^3x \\ &= \frac{4\pi\xi}{g}M_H, \end{aligned} \quad (10)$$

when the vacuum expectation value of the Higgs field is non-zero. Hence the minimum dimensionless total energy is M_H and general non BPS solutions must possess energy $E \geq M_H$.

The magnetic ansatz used for constructing various MAP configurations (Kleihaus & Kunz 2000) is,

$$
\begin{aligned}
A_i^a &= -\frac{1}{r}\psi_1\hat{u}_\phi^a\hat{\theta}_i + \frac{1}{r}\psi_2\hat{u}_\theta^a\hat{\phi}_i \\
&\quad + \frac{1}{r}R_1\hat{u}_\phi^a\hat{r}_i - \frac{1}{r}R_2\hat{u}_r^a\hat{\phi}_i, \\
\Phi^a &= \Phi_1\hat{u}_r^a + \Phi_2\hat{u}_\theta^a,
\end{aligned} \tag{11}
$$

where ψ_1, ψ_2, R_1, R_2, Φ_1 and Φ_2 are functions of r and θ. The generalized asymptotic solution that includes the Jacobi elliptic functions are given as follow:

$$
\begin{aligned}
\psi_1 &= 1 - q + p\left\{\frac{\cos q\theta\ \partial_\mu E_2(u,k) - \cos q\theta\ \partial_\mu E_1(u,k)}{\sin q\theta\ E_2(u,k) + \cos q\theta\ E_1(u,k)}\right\}, \\
\psi_2 &= \frac{n\left\{\sin\theta + \left\{\cos q\theta\ E_2(u,k) - \sin q\theta\ E_1(u,k)\right\}(a\cos\theta + b)\right\}}{\sin\theta}, \\
R_2 &= \frac{n\left\{\cos\theta - \left\{\sin q\theta\ E_2(u,k) + \cos q\theta\ E_1(u,k)\right\}(a\cos\theta + b)\right\}}{\sin\theta}, \\
R_1 &= 0, \\
\Phi_1 &= \xi\left\{\sin q\theta\ E_2(u,k) + \cos q\theta\ E_1(u,k)\right\}, \\
\Phi_2 &= \xi\left\{\cos q\theta\ E_2(u,k) - \sin q\theta\ E_1(u,k)\right\},
\end{aligned} \tag{12}
$$

where $u = p\theta$ and $E_1(u, k)$ and $E_2(u, k)$ are a pair of non-singular Jacobi elliptic functions that satisfy the relation $E_1(u, k)^2 + E_2(u, k)^2 = 1$ and one of the possible pair of non-singular Jacobi elliptic functions (termed as JEA) are $E_1(u, k) = cn(u, k)$, $E_2(u, k) = sn(u, k)$, where $0 \leq k \leq 1$ is the Jacobi elliptic parameter and $p = p(k, q)$ is given by:

$$
p(k,q) = \frac{2(q+1)}{\pi}\int_0^{\pi/2}\frac{d\theta}{\sqrt{1 - k^2\sin^2\theta}}, \tag{13}
$$

where $0 \leq k \leq 1$ and q is any real number. When $k = 0$, $p(0, q) = 1 + q$, the numerical solution with $\psi_1 = \psi_2 = 2$, $R_1 = R_2 = 0$, $\Phi_1 = \varepsilon\cos\theta$, and $\Phi_2 = \xi\sin\theta$ at large is just the standard 1-MAP solution. In Teh et al. (2012), the Jacobi elliptic 1-MAP solutions are solved numerically for four different sets of parameters $(p, q) = (1, -\frac{1}{2})$, $(2, 0)$, $(3, \frac{1}{2})$, $(4, 1)$, with $k = k_1 = 0.9844325133$, the Higgs field self-coupling constant $\lambda = 0$ and 1, and the winding numbers $n = 1,2,3,4,5$, and 6. In this paper, we study the decomposition of the Jacobi elliptic 1-MAP solutions with $n = 1$ only.

CHO ABELIAN DECOMPOSITION

From the work in Cho (1980, 1981) and Cho et al. (2008), it is possible to decompose the gauge potential into the restricted potential and the valence part,

$$
\begin{aligned}
&A_i^a = \hat{A}_i^a + X_i^a = A_i\hat{n}^a - \frac{1}{g}\varepsilon^{abc}\hat{n}^b\partial_i\hat{n}^c + X_i^a, \text{ where} \\
&\hat{n}^a\hat{n}^a = 1, \hat{n}^a X_i^a = 0, A_i = A\hat{\phi}_i.
\end{aligned} \tag{14}
$$

The decomposition (14) introduces the idea of dual structure, where the restricted potential generates the field strength decomposition,

$$
\begin{aligned}
&\hat{F}_{ij}^a = \left(F_{ij} + H_{ij}\right)\hat{n}^a, \text{ with } F_{ij} = \partial_i A_j - \partial_j A_i \text{ and} \\
&H_{ij} = -\frac{1}{g}\varepsilon^{abc}\hat{n}^b\partial_i\hat{n}^c = \partial_i\tilde{C}_j - \partial_j\tilde{C}_i,
\end{aligned}
$$

where $\tilde{C}_i$ is the so-called 'magnetic' potential. It is obvious that the decomposed field strength actually exhibits similar structure as in (4). The advantage of writing the decomposed field strength Abelian structure of the non-Abelian theory is that one can easily obtain and study the built-in Abelian structure.

With the decomposition (14), the field strength tensor can be written as:

$$
F_{ij}^a = \hat{F}_{ij}^a + \hat{D}_i X_j^a - \hat{D}_j X_i^a + g\varepsilon^{abc}X_i^b X_j^c, \tag{16}
$$

The valence potential is written in the form of:

$$
\begin{aligned}
&X_i^a = X_i^1\hat{n}_1^a + X_i^2\hat{n}_2^a, \text{ with } X_i^1 = X_1(r,\theta)\hat{\phi}_i \text{ and} \\
&X_i^2 = X_3(r,\theta)\hat{r}_i + X_4(r,\theta)\hat{\theta}_i,
\end{aligned} \tag{17}
$$

and the orthonormal unit vectors are defined as:

$$
\begin{aligned}
\hat{n}^a &= \sin\alpha(r,\theta)\cos n\phi\ \delta_1^a + \sin\alpha(r,\theta)\sin n\phi\ \delta_2^a + \cos\alpha(r,\theta)\delta_3^a, \\
\hat{n}_1^a &= \cos\alpha(r,\theta)\cos n\phi\ \delta_1^a + \cos\alpha(r,\theta)\sin n\phi\ \delta_2^a - \sin\alpha(r,\theta)\delta_3^a, \\
\hat{n}_2^a &= -\sin n\phi\ \delta_1^a + \cos n\phi\ \delta_2^a.
\end{aligned} \tag{18}
$$

Since our model contain Higgs field as the symmetry breaking component, the Higgs field can be written as $\Phi^a = H(r, \theta)\hat{n}^a$ where $H(r, \theta)$ is the modulus of the Higgs field. The orthonormal unit vectors in (18) can be written more specifically as $\hat{n}^a = h_1\hat{u}_r^a + h_2\hat{u}_\theta^a$, $\hat{n}_1^a = -h_2\hat{u}_r^a + h_1\hat{u}_\theta^a$, $\hat{n}_2^a = \hat{u}_\phi^a$, where h_1 and h_2 are defined as $h_1 = \Phi_1/\sqrt{\Phi_1^2 + \Phi_2^2}$, $h_2 = \Phi_2/\sqrt{\Phi_1^2 + \Phi_2^2}$. Hence the field strength tensor can now be written in a more transparent form:

$$
\begin{aligned}
F_{ij}^a &= \left(\hat{\theta}_i\hat{\phi}_j - \hat{\theta}_j\hat{\phi}_i\right)\left\{\hat{n}^a\left(\frac{1}{r}\partial_\theta A + \frac{1}{r}A\cot\theta + \frac{n\partial_\theta\cos\alpha}{r^2\sin\theta} - X_1X_4\right)\right. \\
&\quad \left. + \hat{n}_1^a\left(\frac{1}{r}\partial_\theta X_1 + \frac{1}{r}X_1\cot\theta + \frac{n\cos\alpha\ X_4}{r\sin\theta} + AX_4\right)\right\} \\
&\quad + \left(\hat{r}\hat{\phi}_j - \hat{r}_j\hat{\phi}_i\right)\left\{\begin{aligned}&\hat{n}^a\left(\partial_r A + \tfrac{1}{r}A + \tfrac{n\partial_r\cos\alpha}{r\sin\theta} - X_1X_3\right) \\ &+\hat{n}_1^a\left(\partial_r X_1 + \tfrac{1}{r}X_1 + \tfrac{n\cos\alpha X_3}{r\sin\theta} + AX_3\right)\end{aligned}\right\} \\
&\quad + \left(\hat{r}_i\hat{\theta}_j - \hat{r}_j\hat{\theta}_i\right)\left\{\hat{n}_2^a\left(-\tfrac{1}{r}\partial_\theta X_3 + \tfrac{1}{r}X_4 + \partial_r X_4\right)\right\},
\end{aligned} \tag{19}
$$

and the covariant derivative of the Higgs field becomes,

$$D_i\Phi^a = \left\{\hat{n}^a\left(\partial_r H\right) + \hat{n}_1^a\left(HX_3\right)\right\}\hat{r}_i + \left\{\hat{n}^a\left(\frac{1}{r}\partial_\theta H\right) + n_1^a\left(HX_4\right)\right\}\hat{\theta}_i + \left\{\hat{n}_2^a\left(-HX_1\right)\right\}\hat{\phi}_i. \quad (20)$$

To calculate the separate contribution of the restricted and valence part to the total energy, we refer to (8) and write the energy density of the restricted part as ED(restricted) and the energy density of the valence part as ED(valence), and the dimensionless energy from the restricted and valence part can then be calculated as:

$$E(restricted) = \frac{g}{2\xi}\int r^2 \sin\theta \; ED(restricted)\, dr d\theta,$$
$$E(valence) = \frac{g}{2\xi}\int r^2 \sin\theta \; ED(valence)\, dr d\theta. \quad (21)$$

To study the contribution of the restricted and valence part to the magnetic charge density, we consider a less singular Abelian magnetic field, $B_i = B_i^a\,\Phi^a/\xi$. From (19), the Abelian magnetic field is calculated as:

$$B_i = \hat{r}_i\left\{H\left(\frac{1}{r}\partial_\theta A + \frac{1}{r}A\cot\theta + \frac{n\partial_\theta\cos\alpha}{r^2\sin\theta} - X_1X_4\right)\right\} + \hat{\theta}_i\left\{H\left(\partial_r A + \frac{1}{r}A + \frac{n\partial_r\cos\alpha}{r\sin\theta} - X_1X_3\right)\right\}, \quad (22)$$

and the magnetic charge density is just $MD = 1/2r^2 \sin\theta\, \partial^i B_i$, in which we can use to plot the magnetic charge density due to the restricted and valence potentials.

From (14), the gauge potential and Higgs field can be written as:

$$A_i^a = A\hat{n}^a\hat{\phi}_i + \left\{-g_1 + X_3\right\}\hat{n}_2^a r_i + \left\{g_2 + X_4\right\}\hat{n}_2^a\hat{\theta}_i + \left\{g_3 + X_1\right\}\hat{n}_1^a\hat{\phi}_i; \qquad \Phi^a = H\hat{n}^a, \quad (23)$$

where g_1, g_2 and g_3 are defined as:

$$g_1 = \frac{\partial_r\cos\alpha}{\sin\alpha}, \quad g_2 = \frac{1}{r}\frac{\partial_\theta\cos\alpha}{\sin\alpha}, \quad g_3 = \frac{n\sin\alpha}{r\sin\theta}, \quad (24)$$

and the functions $\cos\alpha$ and $\sin\alpha$ are given by $\cos\alpha = h_1\cos\theta - h_2\sin\theta$, $\sin\alpha = h_1\cos\theta + h_2\sin\theta$. Equation (23) can then be written more specifically as:

$$A_i^a = \left\{g_2 + X_4\right\}\hat{u}_\phi^a\hat{\theta}_i + \left\{Ah + g_3 g\right\}\hat{u}_\theta^a\hat{\phi}_i + \left\{-g_1 + X_3\right\}\hat{u}_\phi^a\hat{r}_i + \left\{Ag - g_3 h - X_1 h\right\}\hat{u}_r^a\hat{\phi}_i,$$
$$\Phi^a = \Phi_1\hat{u}_r^a + \Phi_2\hat{u}_\theta^a, \quad (25)$$

and by considering $A_i = \hat{\Phi}^a A_i^a = A\hat{\phi}_i$, the function A is given by $A = 1/r\,(\psi_2 h_2 - R_2 h_1)$. Hence by comparing (11) and (25), we can deduce that,

$$X_1 = \frac{1}{r}\left\{\psi_2 h_1 + R_2 h_2 - \frac{n}{r}\left(h_1 + h_2\cot\theta\right)\right\},$$
$$X_3 = \frac{R_1}{r} - \frac{\partial_r h_1}{h_3},$$
$$X_4 = -\frac{\psi_1}{r} + \frac{1}{r}\left(1 - \frac{\partial_\theta h_1}{h_2}\right), \quad (26)$$

From (25), we can write the restricted potential as:

$$A_i^a(restricted) = \left\{g_2\right\}\hat{u}_\phi^a\hat{\theta}_i + \left\{Ah_2 + g_3 h_1\right\}\hat{u}_\theta^a\hat{\phi}_i + \left\{-g_1\right\}\hat{u}_\phi^a\hat{r}_i + \left\{Ah_1 - g_3 h_2\right\}\hat{u}_r^a\hat{\phi}_i, \quad (27)$$

whereas the valence potential can be written as:

$$A_i^a(valence) = \left\{X_4\right\}\hat{u}_\phi^a\hat{\theta}_i + \left\{-X_1 h_1\right\}\hat{u}_\theta^a\hat{\phi}_i + \left\{X_3\right\}\hat{u}_\phi^a\hat{r}_i + \left\{-X_1 h_2\right\}\hat{u}_r^a\hat{\phi}_i, \quad (28)$$

In Figure 1, we show the profile functions for the restricted, valence and original part of the gauge potential functions ψ_1 for standard MAP solution, other functions of ψ_2, R_1, R_2 exhibit similar behaviour.

From (10) and (21), we manage to calculate the separate contribution of the restricted and valence energy density part to the total energy and we tabulate the data in Table 1, where we show the values of E(restricted), E(valence) and E(total) for the standard MAP and JEA MAP solutions with $(p, q) = (1, -\frac{1}{2}), (2, 0), (3, \frac{1}{2}), (4, 1)$ and $k =$

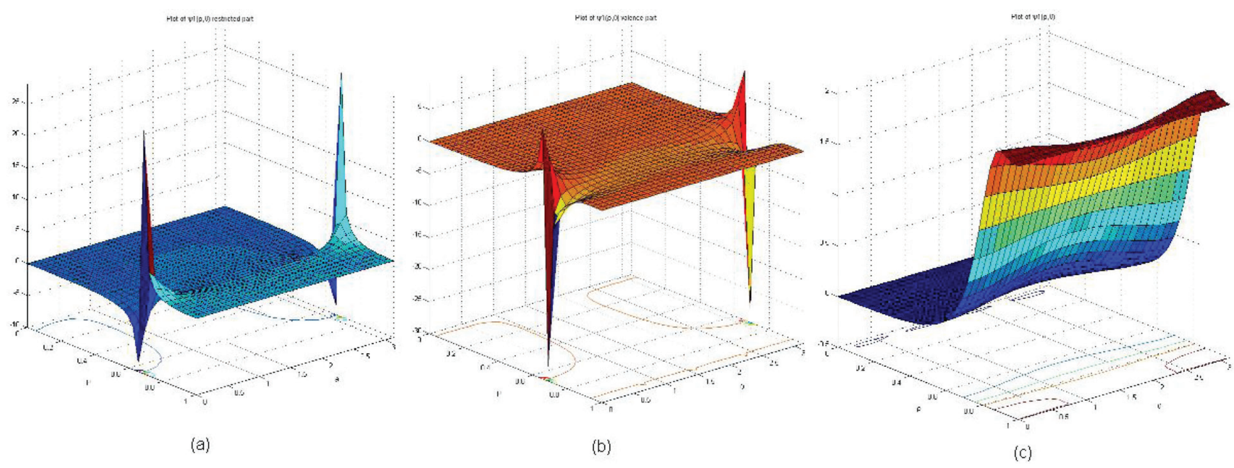

FIGURE 1. Profile for (a) restricted (b) valence and (c) original part of function for the Standard MAP solution

TABLE 1. Table of the dimensionless restricted part energy, E(restricted); valence part energy, E(valence) and total energy E(total) of the standard MAP and Jacobi elliptic 1-MAP solutions for $\lambda = 0,1$ and $n = 1$

$\lambda = 0$	Std MAP	JEA(1,-1/2)	JEA(2,0)	JEA(3,1/2)	JEA(4,1)
E(restricted)	16.156	9.0573	16.0658	19.5007	16.9864
E(valence)	-14.4643	-7.3668	-14.3772	-17.8189	-15.3108
E(total)	1.6917	1.6905	1.6885	1.6818	1.6756
$\lambda = 1$	Std MAP	JEA(1,-1/2)	JEA(2,0)	JEA(3,1/2)	JEA(4,1)
E(restricted)	21.1801	5.0731	21.1027	28.0064	21.2362
E(valence)	-18.8057	-2.7442	-18.7281	-25.6204	-18.8593
E(total)	2.3745	2.329	2.3746	2.386	2.3769

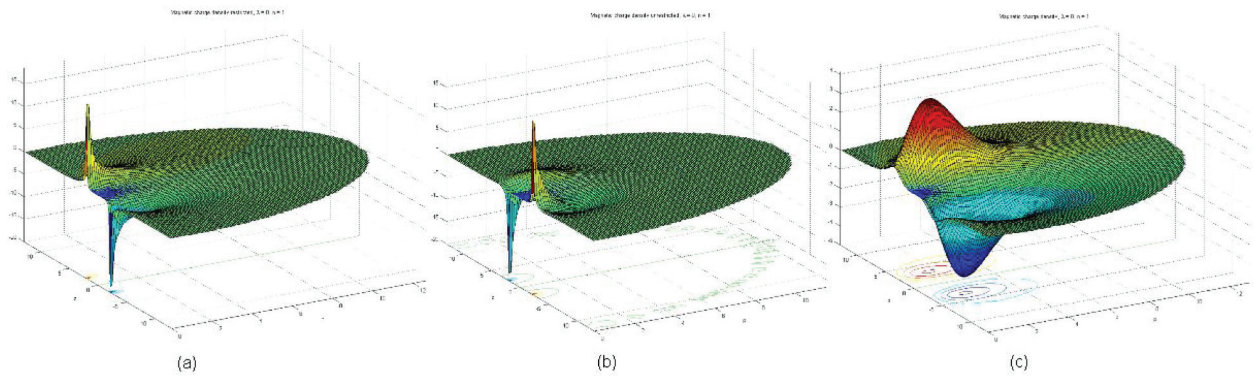

FIGURE 2. Profile for (a) restricted, (b) valence and (c) original part of the magnetic charge density

$k_1 = 0.9844325133$, the Higgs field self-coupling constant $\xi = 1$ and the winding numbers $n = 1$. From Table 1, we can conclude that the valence potential has served to smooth out the original singularities in the restricted part by producing a negative energy density that will lead to a finite total energy (Figure 2).

CONCLUSION

To conclude, our work has shown that the role of the valence part is to smooth out the singularity carried by the restricted potential at the location of the monopole (antimonopole). Hence, the singularity that is present in the energy and magnetic charge density, is also removed by the valence potential. The values of energy and total magnetic charge contributed by the restricted and valence potential are tabulated in Table 1. The technique of decomposing other magnetic monopoles solutions is straight forward and further work to decompose the gauge potential of the finite energy one-half monopole solution (Ng et al. 2012; Teh et al. 2012) will be reported elsewhere.

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